Solution to the November, 2019 Challenge, M&m&4M*

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Physics Challenge for Teachers and Students

Solution to the November, 2019 Challenge, *M&m&4M*

The double Atwood machine can be solved efficiently using the Lagrangian formulation of classical mechanics, but because many readers will have only seen Newton’s laws, we will use the latter.

Newton’s second law applied to the block of mass $m$ gives (with the acceleration convention in the free-body diagrams)

$$T_1 - w_1 = ma_1 \implies T_1 - mg = ma_1$$

Newton’s second law applied to the lower massless pulley gives

$$T_1 - 2T_2 = 0(-a_1) \implies T_1 = 2T_2$$

where it’s clear that if $m$ accelerates upwards with magnitude $a_1$, then the lower pulley must accelerate downwards with magnitude $a_1$.

Newton’s second law applied to the block of mass $4M$ gives

$$T_2 - W_3 = (4M)(-a_2) \implies T_2 - 4Mg = -4Ma_2$$

Newton’s second law applied to the block of mass $M$ gives

$$T_2 - W_2 = Ma_3 \implies T_2 - Mg = Ma_3$$

Now we need to find the relationship between the accelerations because $a_2 \neq a_3$. If we call the acceleration of $4M$ with respect to the center of the lower pulley $-a_r$, then the acceleration of $M$ with respect to the center of the lower pulley is $+a_r$. 


In the frame of reference where the lower pulley is accelerating downward with magnitude \(a_1\):

\[
a_2 = a_1 + a_r
\]

and

\[
a_3 = a_r - a_1 \implies 2a_1 = a_2 - a_3
\]

Now we just solve for the five unknowns \(T_1, T_2, a_1, a_2,\) and \(a_3\) using the five equations above. The solutions are:

\[
T_1 = \frac{4mg}{2 + \frac{5m}{8M}}
\]

\[
T_2 = \frac{2mg}{2 + \frac{5m}{8M}}
\]

\[
a_1 = \frac{2 - \frac{5m}{8M}}{2 + \frac{5m}{8M}} g
\]

\[
a_2 = \frac{2 + \frac{1m}{8M}}{2 + \frac{5m}{8M}} g
\]

\[
a_3 = \frac{11m}{8M} - 2 \frac{11m}{2 + \frac{5m}{8M}} g
\]

Since the numerator of \(a_2\) can never be zero, the block of mass \(4M\) can never be at rest. However, the block of mass \(m\) can be at rest if

\[
a_1 = 0 \implies 2 - 5m/8M = 0 \implies \frac{m}{M} = \frac{16}{5}
\]

And the block of mass \(M\) can be at rest if

\[
a_3 = 0 \implies 11m/8M - 2 = 0 \implies \frac{m}{M} = \frac{16}{11}
\]

For an interactive double Atwood machine in which one can try mass ratios of the two solutions obtained, see

https://demonstrations.wolfram.com/DoubleAtwoodMachine/
In addition, we recognize the following contributors:

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**Guidelines for contributors**
- We ask that all solutions, preferably in Word format, be submitted to the dedicated email address challenges@aapt.org. Each message will receive an automatic acknowledgment.
- If your name is—for instance— Cristiano Messi, please name the file “Messi20Feb” (do not include your first initial) when submitting the February 2020 solution.
- The subject line of each message should be the same as the name of the solution file.
- The deadline for submitting the solutions is the last day of the corresponding month.
- Each month, a representative selection of the successful solvers’ names will be published in print and on the web.
- If you have a message for the Column Editor, you may contact him at korsunbo@post.harvard.edu however, please do not send your solutions to this address.

Many thanks to all contributors and we hope to hear from many more of you in the future!

**Note:** as always, we would very much appreciate reader-contributed original Challenges.

Many thanks to all contributors; we hope to hear from many more of you in the future. We also hope to see more submissions of the original problems – thank you in advance!

*Adapted from Kvant, 2002, 2.*